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THE DIFFRACTION OF PLANE GRAVITATIONAL WAVES BY THE EDGE OF AN ICE COVER*

R.V. GOL'DSHTEIN and A.V. MARCHENKO

The diffraction of plane surface gravitational waves by the edge of an ice cover lying on the surface of an incompressible fluid of infinitely great depth is considered. The ice cover is simulated by a thin elastic plate. The wave reflection and transmission coefficients are determined when it interacts with the ice cover. A wave field is constructed in the fluid under the conditions that a periodic lumped force and a lumped moment act on the edge of the ice cover. It is shown that as the incident wavelength increases the reflection coefficient tends to zero and the transmission coefficient tends to unity.

Problems of hydroacoustic wave scattering by inhomogeneities of a thin elastic plate covering the whole surface of a fluid were investigated in /1-4/. The problem of wave diffraction by the edge of ice cover lying on a stratified fluid surface, as simulated by a solid wing has been examined in /5/. The diffraction of surface waves from the junction of two ice fields of different thickness was investigated by the Peters method in /6/. It was shown that as the incident wavelength increases the reflection coefficient tends to zero and the transmission coefficient tends to unity.

1. Let us consider the potential motion of a heavy incompressible fluid of great depth situated under an ice cover for $x > 0$. The fluid particle velocity potential is denoted by φ.

In the stationary case the ice cover is submerged in the fluid at a depth h that can be determined from Archimedes law: $h = h_0\rho_i/\rho_w$, where ρ_i and ρ_w are the densities of the ice and the fluid, and h_0 is the thickness of the ice. We draw the horizontal x axis along the lower surface of the ice cover in the rest state. We draw the y axis vertically downward Fig.1). Let η be the deflection of the fluid free surface from the horizontal equilibrium position.

We introduce the dimensionless variables (marked with primes)

$$\varphi' = \varphi(a\sqrt{g\lambda})^{-1},\ x' = x\lambda^{-1},\ y' = y\lambda^{-1},\ t' = t\sqrt{g/\lambda}$$

where a is the incident wave amplitude, and λ divided by 2π is the incident wave length.

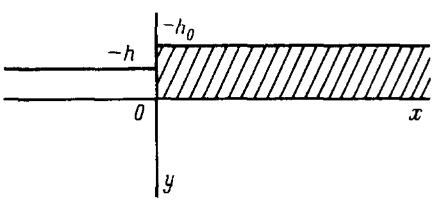

Fig.1

The complete system of equations with linearized boundary conditions that describes the process under consideration has the following form in dimensionless variables (the primes are henceforth omitted)

$$\varphi_{xx} + \varphi_{yy} = 0;\ x < 0,\ -\Delta < y;\ x > 0,\ 0 < y,\ \Delta \equiv h\lambda^{-1} \tag{1.1}$$

$$p = p_a = \text{const};\ x < 0,\ y = -\Delta \tag{1.2}$$

$$\eta_t = \varphi_y;\ x < 0,\ y = -\Delta;\ x > 0,\ y = 0 \tag{1.3}$$

$$p = p_i + p_a;\ x > 0,\ y = 0 \tag{1.4}$$

$$\varphi \to 0;\ y \to \infty \tag{1.5}$$

Here p is the pressure in the fluid, p_a is atmospheric pressure, and p_i is the additional pressure due to the ice cover.

We write the Cauchy-Lagrange integral in the form

$$\varepsilon\varphi_t - \varepsilon y + p/(\rho_w g\lambda) + \Delta = f(t),\ \varepsilon \equiv a\lambda^{-1} \tag{1.6}$$

Setting $f(t) - p_a/(\rho_w g\lambda) + \Delta$, and using (1.6), we can rewrite the boundary conditions (1.2) and (1.3) in the form

$$\varphi_{tt} - \varphi_y = 0,\ y = -\Delta,\ x < 0 \tag{1.7}$$

The additional pressure induced by the ice cover in the fluid is represented in the form $p_i = p_u + \rho_i h_0 g$, where p_u is the pressure due to the elasticity of the ice. The pressure p_u can be determined from the equation of the vibrations of a thin elastic plate /7/ by which the ice cover is simulated in this problem

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$$\frac{p_u}{\rho_w g\lambda} = -\varepsilon\Delta\eta_{tt} - \varepsilon D\eta_{xxxx}, \quad D = \frac{Eh_0^3}{12(1-\nu^2)\rho_w g\lambda^4} \tag{1.8}$$

Here E and ν is Young's modulus and Poisson's ratio for ice.

Using (1.8), the boundary conditions (1.3) and (1.4) can be written in the form

$$\varphi_{tt} - \varphi_y - \Delta\varphi_{ytt} - D\varphi_{yxxxx} = 0; \quad x > 0, \quad y = 0 \tag{1.9}$$

Henceforth, waves will be considered that are long compared with the thickness of the ice: consequently we set $\Delta = 0$. It is assumed that the dependence of all the functions on time is expressed in dimensionless form by the factor e^{it}. Taking the above into account, as well as relationships (1.7) and (1.9), system (1.1)-(1.5) can be written in the form

$$\varphi_{xx} + \varphi_{yy} = 0; \ -\infty < x < \infty, \ y > 0 \tag{1.10}$$

$$\varphi + \varphi_y = 0; \ -\infty < x < 0, \ y = 0 \tag{1.11}$$

$$\varphi + \varphi_y + D\varphi_{yxxxx} = 0; \ 0 < x < \infty, \ y = 0 \tag{1.12}$$

$$\varphi \to 0; \ y \to \infty \tag{1.13}$$

As will be seen subsequently, the exact solution of problem (1.10)-(1.13) is not unique and depends on the four constants $A_\pm$, $k_\pm$.

Three problems are examined:

a) the diffraction of plane waves by the edge of an ice cover; the wave arrives from the pure water side $(x < 0)$;

b) the diffraction of plane waves by the edge of ice cover; the wave arrives from the fluid beneath the ice $(x > 0)$;

c) determination of the fluid motion if periodic concentrated forces and moments act on the edge of the ice cover.

In dimensionless form the concentrated forces and moments are defined by the following formulas /1/

$$f = -i\varphi_{yxxx}, \ m = -i\varphi_{yxx}$$

We take as characteristic quantities of the forces and moments

$$D_f = \frac{Eh_0^3 a}{12(1-\nu^2)\lambda^3}, \quad D_m = \frac{Eh_0^3 a}{12(1-\nu^2)\lambda^2}$$

The constants $A_\pm$, $k_\pm$ in problems a) and b) are determined from the conditions

$$\varphi \to e^{-(ix+y)} + R_1 e^{ix-y}, \quad x \to -\infty \tag{1.14}$$

$$\varphi \to e^{k(ix-y)} + R_2 e^{-k(ix+y)}, \quad x \to +\infty$$

$$\varphi \to T_1 e^{-k(ix+y)}, \quad x \to +\infty; \tag{1.15}$$

$$\varphi \to T_2 e^{ix-y}, \quad x \to -\infty$$

$$f \to 0, \ m \to 0; \ x \to +0, \ y \to 0 \tag{1.16}$$

$R_{1,2}$ and $T_{1,2}$ are the wave reflection and transmission coefficients.

The constants $A_\pm$, $k_\pm$ in problem c) are determined from the conditions

$$\varphi \to R_3 e^{-k(ix+y)}, \quad x \to +\infty; \quad \varphi \to T_2 e^{ix-y}, \quad x \to -\infty \tag{1.17}$$

$$f \to -iF', \ m \to -iM'; \ x \to +0, \ y \to 0 \tag{1.18}$$

M' and F' are related to the dimensional quantities of the effective forces and moments by the formulas $M = -iM' D_m$ and $F = -iF' D_f$.

2. We will seek the solution of the problems under consideration in the form

$$\varphi = \frac{1}{2\pi i}\int_{-\infty}^{\infty} p(k)\, e^{ikx - |k| y}\, dk \tag{2.1}$$

The solution in the form (2.1) satisfies (1.10) and condition (1.13) identically. Consequently, there are no sources and sinks in the whole domain of motion, and the integral (2.1) should converge at each point of the half-space $y > 0$. We hence have

$$p(k) < O(k^{-2}), \ |k| \to \infty \tag{2.2}$$

The contour of integration in (2.1) can be deformed into the domain of analyticity of the function $p(k)$ in the plane k. Substituting (2.1) into (1.11) and (1.12), we obtain

$$\int_{L_1} p(k)(1-|k|)e^{ikx}\,dk = 0; \quad x<0, \quad \operatorname{Im} k \leqslant 0 \tag{2.3}$$

$$\int_{L_2} p(k)(1-|k|(1+Dk^4))e^{ikx}\,dk = 0; \quad x>0, \quad \operatorname{Im} k \geqslant 0 \tag{2.4}$$

The curves L_1 and L_2 lie, respectively, in the lower and upper half-planes k. Note that if the limits of integration in (2.3) and (2.4) were to remain equal to $\pm\infty$ as in (2.1), then integrals (2.3) and (2.4) would diverge in the ordinary sense. To obtain (2.3), the contour of integration in (2.1) must first be deformed in the lower k half-plane in such a manner that the ends of L_1 would approach an infinitely remote point in directions different from the real axis, and then φ would be substituted into the boundary condition (1.11). An analogous procedure must be carried out in the upper k half-plane to obtain (2.4).

When going over to the complex k plane the function $|k|$ must be determined. Let us define it as the limit as $\varepsilon \to 0$ for functions $\varkappa_\pm$ that are analytic in a two-sheeted Riemann surface: $\varkappa_\pm = \sqrt{k^2 \pm i\varepsilon^2}$, where the sheet of the Riemann surface is selected on which $\operatorname{Re} \varkappa_\pm >$ 0 on the real axis. Let us use the notation $|k|_\pm = \lim \varkappa_\pm$ as $\varepsilon \to 0$. We note that the functions $|k|_\pm$ can take different values in the plane k, however, they certainly equal $|k|$ on the real axis.

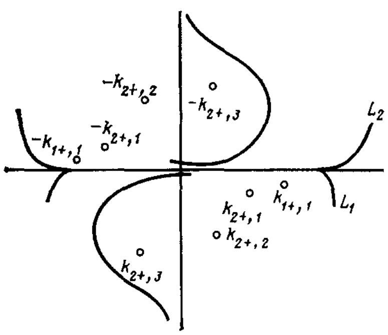

Fig.2

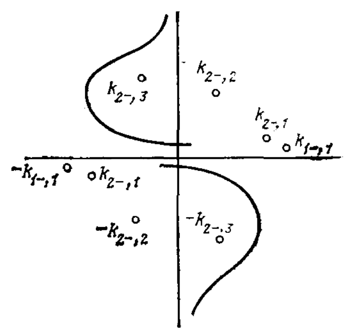

Fig.3

Eqs.(2.3) and (2.4) will be satisfied identically if it is assumed that

$$p(k)(|k|_\pm - 1) \equiv \Phi_-(k), \quad p(k)(|k|_\pm(k^4 + D^{-1}) - D^{-1}) \equiv \Phi_+(k)$$

where Φ_+ and Φ_- are functions analytic in the upper and lower k half-planes, respectively. At infinity $\Phi_\pm$ can have singularities of finite-order pole type. On the real k axis

$$\Phi_+/\Phi_- = [|k|(k^4 + D^{-1}) - D^{-1}]/(|k| - 1) \tag{2.5}$$

To find Φ_+ and Φ_- we factorize the functions on the right side of (2.5).

The functions $G_{1\pm} \equiv \varkappa_\pm(\varkappa_\pm - 1)$, $G_{2\pm} \equiv \varkappa_\pm^2(\varkappa_\pm^4 + D^{-1}) - D^{-1}\varkappa_\pm$ have 4 and 12 roots, respectively, in the complex k plane. There is a root of opposite sign for each root. Each of the functions $G_{i\pm}$ has roots $k = \pm\varepsilon e^{\pm i\pi/4}$, that lie on slits of the Riemann surface and will henceforth not be considered.

To determine $\varkappa_+$ we split the k plane as shown in Fig.2, and to determine $\varkappa_-$ we split it as shown in Fig.3. By using the Routh-Hurwitz criterion the location of the roots $G_{i\pm}$ in the k plane can easily be established as $\varepsilon \to 0$. Namely, G_{1+} has two roots $\pm k_{1+,1}$ close to ± 1 and lying in the second and fourth quadrants, while G_1 has two roots $\pm k_{1-,1}$ close to ± 1 and lying in the first and third quadrants. The equality $k_{1\pm,1} = 1$ is satisfied for $\varepsilon = 0$. The function G_{2+} has the roots $\pm k_{2+,1}$ in the second and fourth quadrants on the sheet under consideration and taking the real values $\pm k_{2,1}$ for $\varepsilon = 0$, and the roots $\pm k_{2+,2}, \pm k_{2+,3}$, whose location is displayed in Fig.2. On the sheet under consideration the function G_{2-} has roots $\pm k_{2-,1}$ lying in the first and third quadrants and taking on the real values $\pm k_{2,1}$ at $\varepsilon = 0$ and the roots $\pm k_{2-,2}, \pm k_{2-,3}$ whose locatión is displayed in Fig.3.

We note that as $D \to 0$

$$k_{2,1} \to 1, \quad k_{2-,2} \to D^{-1}e^{i\pi/4}, \quad k_{2-,3} \to -k_{2+,2}$$

We introduce the function

$$g_{i\pm}=G_{i\pm}/\Pi_{i\pm},\quad \Pi_{1\pm}=k^2-k_{1\pm,1}^2$$
$$\Pi_{2\pm}=(k^2-k_{2\pm,1}^2)(k^2-k_{2\pm,2}^2)(k^2-k_{2\pm,3}^2)$$

In the limiting case $\varepsilon=0$ the functions $g_{i\pm}$ have no zeros on the sheet under consideration, are bounded and tend to unity at infinity. These functions can be factorized as follows /8/

$$g_{i\pm}=g_{i\pm}^{+}g_{i\pm}^{-},\quad g_{i\pm}^{\pm}(k)=\exp\left[\pm\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln g_{i\pm}(\lambda)}{\lambda-k}d\lambda\right] \tag{2.6}$$

Here $g_{i\pm}^{+}$, $g_{i\pm}^{-}$ are functions analytic, respectively, in the upper and lower k half-planes. According to the Sokhotskii theorem, we have on the real axis

$$g_{i\pm}^{\pm}(k)=\exp\left[\pm\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln g_{i\pm}(\lambda)}{\lambda-k}d\lambda\right]G(k) \tag{2.7}$$
$$G(k)=\sqrt{g_{i\pm}(k)},\quad k=\operatorname{Re}k\pm i0$$

It follows from (2.6) that

$$\frac{g_{1\pm}^{\pm}}{g_{2\pm}^{\pm}}=\exp\left[\pm\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln[g_{1\pm}(\lambda)/g_{2\pm}(\lambda)]}{\lambda-k}d\lambda\right]. \tag{2.8}$$

Let us examine the limiting case $D\to 0$. Substituting the asymptotic value of the roots into the definition of $g_{i\pm}$ we obtain that $g_{1\pm}\to g_{2\pm}$ as $D\to 0$ and, therefore

$$g_{1\pm}^{\pm}\to g_{2\pm}^{\pm},\quad D\to 0.$$

We rewrite (2.5) in the form

$$\frac{\Phi_{+}\Pi_{1\pm}^{+}g_{1\pm}^{+}}{\Pi_{2\pm}^{+}g_{2\pm}^{+}}=\frac{\Phi_{-}\Pi_{2\pm}^{-}g_{2\pm}^{-}}{\Pi_{1\pm}^{-}g_{1\pm}^{+}}\equiv F_{\pm} \tag{2.9}$$
$$\Pi_{1+}^{\pm}=k\mp k_{1+,1},\quad \Pi_{2+}^{\pm}=(k\mp k_{2+,1})(k\mp k_{2+,2})(k\mp k_{2+,3})\,\Pi_{1-}^{\pm}=k\pm k_{1-,1}$$
$$\Pi_{2-}^{\pm}=(k\pm k_{2-,1})(k\pm k_{2-,2})(k\pm k_{2-,3})$$

It follows from (2.2) and (2.9) that $F_{\pm}$ are analytic in the whole plane k and the condition $F_{\pm}\leqslant O(k)$ is satisfied as $|k|\to\infty$. According to Liouville's theorem /9/, the functions $F_{\pm}$ are first-order polynomials

$$F_{+}\equiv A_{+}(k-k_{+}),\quad F_{-}\equiv A_{-}(k-k_{-})$$

It follows from (2.1) and the first definition of $\Phi_{\pm}$ that the solution of system (1.10)-(1.13) can be written in the form

$$\varphi=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\left[\frac{F_{+}|k|_{+}\exp(-|k|_{+}y)}{\Pi_{1+}^{+}\Pi_{2+}^{-}g_{1+}^{+}g_{2+}^{-}}+\frac{F_{-}|k|_{-}\exp(-|k|_{-}y)}{\Pi_{1-}^{+}\Pi_{2-}^{-}g_{1-}^{+}g_{2-}^{-}}\right]\exp(ikx)\,dk \tag{2.10}$$

Formula (2.10) determines the continuous function φ that is bounded in the domain $x\in(-\infty,\infty)$, $y>0$ and depends on the four constants $A_{\pm}, k_{\pm}$.

3. To analyse the solution (2.10) and determine the constants $A_{\pm}$, $k_{\pm}$ from the conditions (1.14)-(1.18), we deform the contour of integration in (2.10) for $x<0$ into the curve L_1, and for $x>0$ into the curve L_2. Taking the residues at the singularities of the integrands, the integrals on L_1 and L_2 can be reduced to integrals over the imaginary half-axes, and (2.12) can be reduced to the form

$$x<0,\ \varphi=-F_{+}(1)\chi_{+}^{-}(1)\exp(ix-y) \tag{3.1}$$
$$F_{-}(-1)\chi_{-}^{-}(-1)\exp(-ix-y)-I_{-}$$
$$x>0,\quad \varphi=-F_{+}(-k_{2-,1})\alpha_1^{+}\exp[k_{2-,1}(-ix-y)]+ \tag{3.2}$$
$$F_{-}(k_{2-,1})\alpha_1^{-}\exp[k_{2-,1}(ix-y)]-[F_{+}(k_{2+,2})+\alpha_2^{+}+$$
$$F_{-}(-k_{2+,2})\alpha_2^{-}]\exp[k_{2+,2}(-ix-y)]+[F_{+}(k_{2-,2})\alpha_3^{+}+F_{-}(k_{2-,2})\alpha_3^{-}]\times$$
$$\exp[k_{2-,2}(ix-y)]+I_{+}$$

Here

$$\alpha_1^{\pm} = \chi_{\pm}^{+}(\mp k_{2-,1})[(k_{2-,1}-k_{2-,2})(k_{2-,1}-k_{2+,2})(k_{2-,1}-k_{2-,3})(k_{2-,1}-k_{2+,3})]^{-1} \tag{3.3}$$
$$\alpha_2^{\pm} = \chi_{\pm}^{+}(-k_{2+,2})[(k_{2-,1}-k_{2+,2})(k_{2-,2}-k_{2+,2})(k_{2-,3}-k_{2+,2})(k_{2+,3}-k_{2+,2})]^{-1}$$

$$\alpha_3^{\pm} = \chi_{\pm}^{+}(k_{2-,2})[(k_{2-,2}-k_{2-,1})(k_{2-,2}-k_{2+,2})(k_{2-,2}-k_{2-,3})(k_{2-,2}-k_{2+,3})]^{-1},$$
$$\chi_{\pm}^{\pm} = g_{1\pm}^{\pm}\Pi_{1\pm}^{\pm}/(g_{2\pm}^{\pm}\Pi_{2\pm}^{\pm})$$
$$I_- = \frac{1}{2\pi i}\int_0^{-i\infty} \frac{F_+\chi_+^- + F_-\chi_-^-}{k^2-1}[(k-1)e^{ky} + (k+1)e^{-ky}]e^{ikx}dk$$
$$I_+ = \frac{D}{2\pi i}\int_0^{i\infty} \frac{F^+\chi_+^+ + F_-\chi_-^+}{k^2(Dk^4+1)^2-1}[(k(Dk^4+1)-1)e^{ky} +$$
$$(k(Dk^4+1)+1)e^{-ky}]e^{ikx}dk$$

We determine the concentrated forces and moments acting on the edge of the ice cover from (3.2)

$$\varphi_{yxx}(y=0, x=+0) = A_+r_1^+ + A_+k_+r_2^+ + A_-r_1^- + A_-k_-r_2^- \tag{3.4}$$
$$\varphi_{yxxx}(y=0, x=+0) = A_+p_1^+ + A_+k_+p_2^+ + A_-p_1^- + A_-k_-p_2^-$$
$$r_1^{\pm} = k_{2-,1}^4\alpha_1^{\pm} + k_{2+,2}^4\alpha_2^{\pm} + k_{2-,2}^4\alpha_3^{\pm} + K_4^{\pm} \tag{3.5}$$
$$r_2^{\pm} = k_{2-,1}^3\alpha_1^{\pm} + k_{2+,2}^3\alpha_2^{\pm} - k_{2-,2}^3\alpha_3^{\pm} - K_3^{\pm}$$
$$p_1^{\pm} = -i[\pm k_{2-,1}^5\alpha_1^{\pm} + k_{2+,2}^5\alpha_2^{\pm} - k_{2-,2}^5\alpha_3^{\pm} - K_5^{\pm}]$$
$$p_2^{\pm} = -i[k_{2-,1}^4\alpha_1^{\pm} + k_{2+,2}^4\alpha_2^{\pm} + k_{2-,2}^4\alpha_3^{\pm} + K_4^{\pm}]$$
$$K_n^{\pm} = \frac{D}{2\pi i}\int_0^{i\infty} k^n J^{\pm}dk, \quad J^{\pm} = -\frac{2\chi_{\pm}^{+}}{k^2(Dk^4+1)-1}$$

Let us consider problems a and b. We have from (1.14), (1.15), (3.1) and (3.2)

$$k_- = k_{2-,1}, \quad A_- = -[(1+k_{2-,1})\chi_-^-(-1)]^{-1} \text{ (problem a)} \tag{3.6}$$
$$R_1 = -A_+(1-k_+)\chi_+^-(1), \quad T_1 = A_+(k_{2-,1}+k_+)\alpha_1^+$$
$$k_- = -1, \quad A_- = [(1+k_{2-,1})\alpha_1^-]^{-1} \text{ (problem b)}$$
$$R_2 = A_+(k_{2-,1}+k_+)\alpha_1^+, \quad T_2 = -A_+(1-k_+)\chi_+^-(1)$$

The constants A_+, k_+ are found from (1.16) and (3.4)

$$A_+ = (p_3r_2^+ - r_3p_2^+)(r_1^+p_2^+ - p_1^+r_2^+)^{-1} \tag{3.7}$$
$$k_+ = (r_1^+p_3 - p_1^+r_3)(r_3p_2^+ - p_3r_2^+)^{-1}$$
$$r_3 \equiv A_-r_1^- + A_-k_-r_2^-, \quad p_3 = A_-p_1^- + A_-k_-p_2^-$$

We will investigate the asymptotic form of formulas (3.6) and (3.7) in the limit case $D \to 0$, which corresponds to considering waves of very long wavelength. Using the asymptotic form of the roots $k_{2-,j}$ and the limiting relationship (2.9) we have $\alpha_j^+ \to \alpha_j^-, J^+ \to J^-$, from (3.3) and (3.5), and consequently, $r_j^+ \to r_j^-, p_j^+ \to p_j^-$ as $D \to 0$. In the limit $D \to 0$ we obtain from (1.16) and (3.4)

$$k_+ \to 1, \ A_+ \to -A_- \ \text{(problem a)} \qquad A_+ \to A_-, \ k_+ \to -1 \ \text{(problem b)}$$

Substituting these values into (3.6), we have $R_{1,2} \to 0$, $T_{1,2} \to 1$. This means that the presence of an ice cover on a fluid surface does not influence the propagation of very long waves.

Let us consider problem c. We have from (1.17), (3.1) and (3.2)

$$A_- = 0, \quad R_3 = A_+(k_{2-,1}+k_+)\alpha_1^+, \quad T_3 = -A_+(1-k_+)\chi_-^+(1)$$

The constants A_+, k_+ are found from (1.18) and (3.4) and are determined by (3.7) in which we must set $r_3 \equiv M'$, $p_3 \equiv F'$.

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HIGH-FREQUENCY ASYMPTOTICS OF ACOUSTIC PRESSURE FOR BOUNDED WAVE BEAM SCATTERING BY AN ELASTIC SPHERE*

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Asymptotic high-freqeuncy estimates are obtained for the amplitudes of specular and non-specular reflections with extraction of the contribution of sound reradiation into the surrounding medium by Rayleigh type surface elastic waves. The conditions are found that govern the magnification of scattering in the opposite direction. The theoretical explanation of the book reflection effect /1/ for bounded sound beam incidence on the plane interface of a fluid-elastic solid is given by many authors in different situations (/12/, say). As for non-specular reflection of a plane sound wave by bounded elastic bodies (plates, cylinders, rods, and shells enclosed in a screen), studied most thoroughly in /3-9/, this effect is a consequence of satisfying the space-time resonance conditions between the incident acoustic wave and the normal surface waves excited in an elastic solid under total internal reflection.

It is interesting to clarify and describe the book reflection of a bounded sound beam incident on the curvilinear interface between two media. Selection of the contributions of surface waves in the echo signal from elastic cylinders was carried out experimentally /10, 11/ by sounding a narrow part of an object surface by a pencil beam near the critical angles of surface acoustic wave excitation. An analytic description of such a process was given in /12/ for analogous wave excitation conditions in the case of spherical and cylindrical elastic reflectors. However, the echo signals reradiated by the surface waves were only examined in the domain of the geometric shadow of the objects. Non-specular reflection in the reverse direction directly from the sounded section of the interfacial boundary without preliminary residency in the shadow domain was not analysed.

1. Let a sound beam, whose effective transverse section near the interface of two media is represented as a narrow circular ring of width ν_i, impinge on an elastic object of spherical shape that is in an ideal compressible fluid. The acoustic pressure of the incident beam is expressed by the formula

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